

TOTAL EMISSIVITY OF CARBON DIOXIDE AND ITS EFFECT ON TROPOSPHERIC TEMPERATURE

Didactic Article

By Nasif Nahle

Scientific Research Director at Biology Cabinet

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Abstract:

By applying generally accepted algorithms on radiative heat transfer, verified through experimentation by Hottel⁽¹⁾, Leckner⁽²⁾ and other contemporary scientists and engineers⁽³⁾⁽⁴⁾⁽⁵⁾, I have found that carbon dioxide molecules possess a low total emissivity at the current density of CO₂ in the atmosphere.

Introduction:

The formula applied by several authors on radiative heat transfer for calculating the atmospheric temperature anomaly caused by carbon dioxide is as follows:

$$\Delta T = \frac{\left(\alpha * \left(\ln \frac{[g_a]_{\infty}}{[g_a]_{std}} \right) \right)}{4 (\sigma)(T)^3}$$

(Formula 1)

Where ΔT is change of temperature, α is climate sensitivity of the absorbent gas (for CO₂ at 1000 K, 1 atm and 0 altitude, it is 5.35 W/m²), $[g_a]_{\infty}$ is instantaneous concentration of the absorbent gas, $[g_a]_{std}$ is the standard concentration of the absorbent gas, σ is the Stefan-Boltzmann constant (5.6697 x 10⁻⁸ W/m² K⁴), and T is the standard temperature (290 K).

Although this formula has been widely used, several factors are missing that have a definitive influence on the results. *Exempli gratia*, it fails to account for the sphericity of the system, the partial pressure of the absorbent gas and its buoyancy.

Another problem with the formula is that it represents homogeneous values for the total emissivity and total absorptivity of the absorbent gas, which is absolutely at odds with what has been observed and verified through experimentation because the total emissivity and total absorptivity of any substance changes in proportion to its partial pressure in any given environment.

The surrounding temperature also imparts a significant influence on the total emissivity and total absorptivity of any substance; the same can be said for the distance which separates the emissive system from the absorbent system. None of these factors are accounted for in the formula derived from the Stefan-Boltzmann principle, which is appropriate when considering blackbodies, but absolutely inappropriate when considering gray bodies at different concentrations in a given medium, such as carbon dioxide in the atmosphere.

The appropriate formula for obtaining the total emissivity of carbon dioxide is as follows:

$$E_{CO_2} = 1 - [(a-1 * 1 - P_E / a + b - (1 + P_E)) * e^{-c (\text{Log}_{10} (p_a L)_m / p_a L)^2}] * (E_{CO_2})_0 \quad (\text{Modest. 2003. Pp. 339-346})_{(4)} \quad (\text{F. 2})$$

$$T = 35 \text{ }^\circ\text{C} = 308 \text{ K}$$

P_E = effective pressure of the absorbent gas $(p + 0.28 p_a) / p_0$.

p = Total pressure of the mixed gases (1 bar for the atmosphere at sea level).

p_a = Partial pressure of the absorbent gas.

$(p_a L)_m$ = Partial pressure of the absorbent gas (a) modified by Planck's function.

$(P_a L)_0$ = partial pressure of the absorbent gas (a) at $L = 0$.

$p_a L$ = Partial pressure of the absorbent gas (a) at $L = 7000 \text{ m}$.

a, b, c = Proportionality constants.

$(E_{CO_2})_0$ = Emissivity of the CO_2 at $(L = 0, p = 1 \text{ bar}, T = 373 \text{ K and } p_{CO_2} L = 0.03 \text{ bar cm}) = 0.0016$. As it can also be iterated from the graphs on the total emissivity of carbon dioxide published in books of Heat Transfer. [*Hottel (1954), Leckner (1972), Pitts & Sissom (1998), Modest (2003), Manrique (2002)*]

Example:

Obtaining the emissivity of CO_2 , the normal intensity of the radiation, the normal intensity of the radiative heat transfer of CO_2 , and the change of temperature caused by the CO_2 if:

- a) The percentage of Carbon Dioxide in the atmosphere is 0.034% (present percentage)
- b) The total pressure of the air is 1.01325 bar (real value)
- c) The current temperature of the surface is 330 K (instantaneous temperature of the local land surface today at 22 hrs UT)
- d) The current temperature of the air is 308 K (instantaneous temperature of the air above the reported surface today at 22 hrs UT)

Obtaining the total emissivity of carbon dioxide

Initially, let us obtain the total emissivity of carbon dioxide at its current concentration in the atmosphere. To do this, we will use the following formula:

$$E_{CO_2} / (E_{CO_2})_0 = 1 - [(a-1 * 1 - P_E / a + b - (1 + P_E)) * e^{-c (\text{Log}_{10} (p_a L)_m / p_a L)^2}]_{(4)} \quad (\text{F. 2})$$

Known magnitudes:

$(E_{CO_2})_0$ at $T = 373$ K and $p = 1$ bar = 0.0016 (2) (See **Formula 3**)

$$t = 308 \text{ K} / 373 \text{ K} = 0.82$$

$$T_0 = 373 \text{ K}$$

$$a = 1 + 0.1 / t^{1.45} = 1 + 0.1 / 0.82^{1.45} = 1.45$$

$$b = 0.23$$

$$c = 1.47$$

$$p_{CO_2} = 0.00034 \text{ bar}$$

$$p_{abs} = 1 \text{ bar}$$

$$p_0 \text{ (absolute pressure)} = 1 \text{ bar}$$

$$P_E = p_{abs} + [0.28 (p_{CO_2})] / p_0 = 1 \text{ bar} + [0.28 (0.00034 \text{ bar})] / 1 = 1.0001 \text{ bar}$$

$$(PCO_2L)_m / (PCO_2L)_0 = 0.225 * t^2 \text{ (if } t > 0.7) = 0.151$$

$$(PCO_2L)_m = (0.225 * t^2) * (PCO_2L)_0 = 0.151 * 0.034 \text{ bar cm} = 0.005134 \text{ bar cm}$$

$$PCO_2L = 0.00034 \text{ bar m} = 0.034 \text{ bar cm}$$

$$T = 308 \text{ K}$$

Emissivity of carbon dioxide at its current concentration in the atmosphere:

FORMULA:

$$E_{CO_2} = \left[1 - \left(\frac{a-1 * 1-PE}{(a+b)-(1+PE)} \right) * \left(e^{-c \left(\text{Log}_{10} \left(\frac{(p_a L)_m}{(p_a L)} \right)^2} \right)} \right) \right] * (E_{CO_2})_0 \quad \text{(Formula 2)}$$

Introducing magnitudes:

$$E_{CO_2} = [1 - ((1.45-1 * 1-1.0001 \text{ bar} / (1.45 + 0.23) - (1 + 1.0001 \text{ bar})) * (e^{-1.47 (\text{Log}_{10} (0.005134 \text{ bar cm} / 0.034 \text{ bar cm}))^2}))] * 0.0016$$

$$E_{(CO_2)} = \left[1 - \left(\frac{(0.45 * -0.0001 \text{ bar})}{((1.68) - (2.0001))} \right) * \left(e^{-1.47 \left(\text{Log}_{10} \left(\frac{(0.005134 \text{ bar cm})}{0.034 \text{ bar cm}} \right)^2} \right)} \right) \right] * 0.0016$$

$$E_{CO_2} = [1 - (((-0.000045 \text{ bar} / -0.3201 \text{ bar})) * e^{-1.47 (\text{Log}_{10} (0.005134 \text{ bar cm} / 0.034 \text{ bar cm})^2}))] * 0.0016$$

$$E_{CO_2} = [1 - (0.00014 * 0.3712)] * 0.0016 = (0.999948) (0.0016) = 0.0017$$

Actually, the total emissivity of carbon dioxide at its current density in the atmosphere is quite low. The total absorptivity of carbon dioxide at its current concentration in the atmosphere is 0.0017.

Therefore, for an air temperature of 308 K (35 °C), carbon dioxide contributes with 13.5 K. The remainder thermal effect of carbon dioxide is exclusively of cooling of the surface and other masses of more efficient absorbent gases.

Another formula for calculating the total emissivity of carbon dioxide is derived from the formula above mentioned, which applies especially at temperatures below 1000 K, e.g. the temperatures of the Earth's atmosphere at different altitudes and partial pressures of carbon dioxide. The formula is as follows:

$$E_{CO_2} = [e (|\sqrt{\text{Log}_{10} (290 \text{ K} * T_{\infty})}| / (-c * 1 \text{ K})) * [p_{CO_2} * 100 / 5 (p_{abs})] \quad (\text{Formula 3})$$

Where T_{∞} is for the instantaneous temperature, c is a proportionality constant (1.47), p_{CO_2} is for the instantaneous concentration of carbon dioxide in the atmosphere, and p_{abs} is for the absolute pressure of the atmosphere.

EXAMPLE:

The proportion of carbon dioxide in the atmosphere is 0.038%, the absolute pressure (p_{abs}) at the sea level is 1 atm, and the instantaneous temperature (T_{∞}) of the air is 32 °C. Calculate the Total Emissivity of carbon dioxide under these given conditions.

Known magnitudes:

Percentage of CO₂ in the atmosphere = 0.038%

$T_{\infty} = 32 \text{ °C} + 273 = 305 \text{ K}$

$c = - 1.47$

$p_{abs} = 1 \text{ atm.}$

$p_{CO_2} = ?$

Before proceeding to the calculation of the total emissivity of carbon dioxide in the current atmosphere, we will proceed to obtain the partial pressure of carbon dioxide in the atmosphere as from its present mass fraction; for doing this, we divide the proportion of carbon dioxide in the atmosphere by 100 and multiply the result by 1 atm:

$p_{CO_2} = (0.038\%/100) * 1 \text{ atm} = 0.00038 \text{ atm.}$

Now, let us proceed to calculate the total emissivity of carbon dioxide at current conditions.

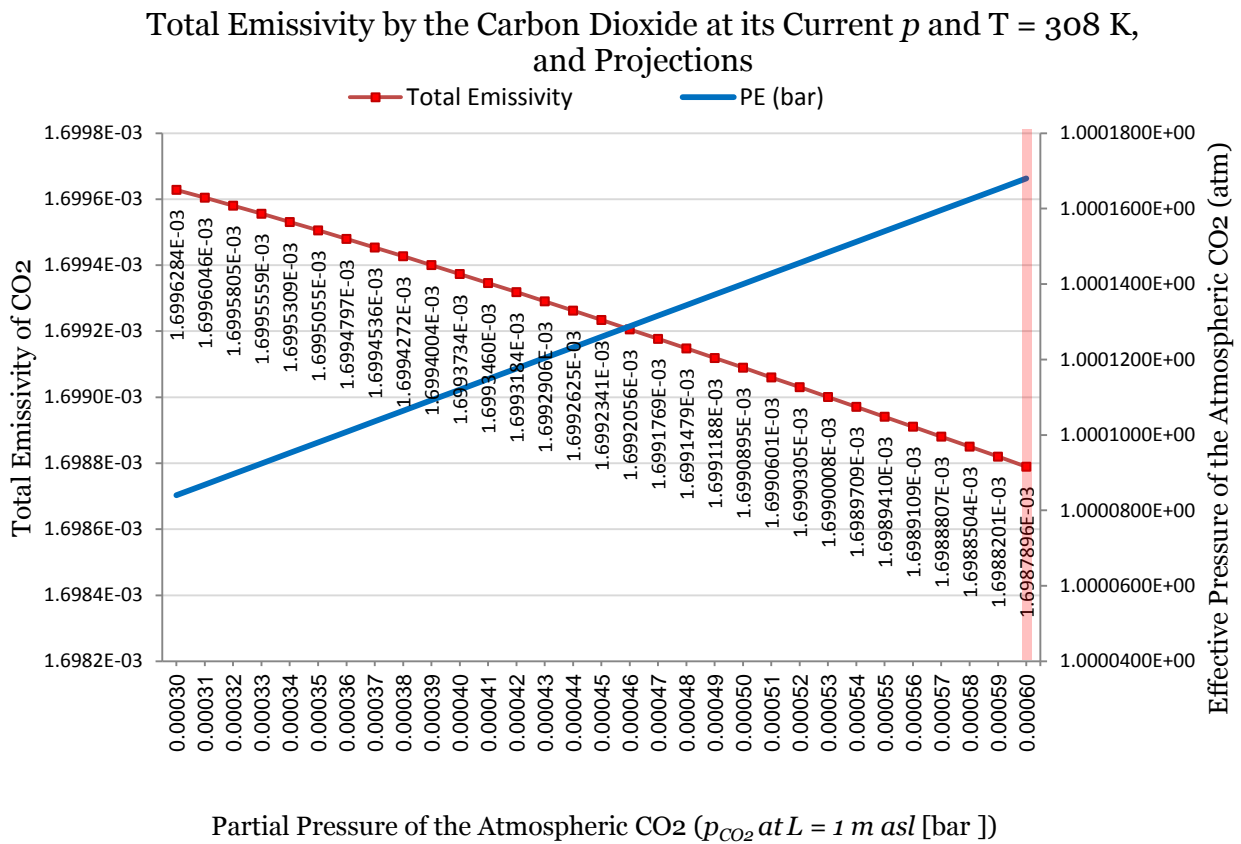
Introducing magnitudes:

$$E_{CO_2} = [e (|\sqrt{\text{Log}_{10} (290 \text{ K} * T_{\infty})}| / (-c * 1 \text{ K})) * (p_{CO_2} * 100 / 5 (p_{abs})) \quad (\text{Formula 3})$$

$$E_{CO_2} = [e ((\sqrt{|\text{Log}_{10} (290 \text{ K} * 305 \text{ K})|}) / (-1.47 * 1 \text{ K})) * (0.00038 \text{ atm} * 100 / 5 (1 \text{ atm}))]$$

$$E_{CO_2} = [e (2.24 \text{ K} / -1.47 \text{ K})] * (0.038 \text{ atm} / 5 \text{ atm}) = (0.218) (0.0076) = 0.0017$$

Something valuable to emphasize is that the total emissivity of carbon dioxide decreases as the density of the gas in the atmosphere increases, as long as the temperature remains constant; as well, when increasing the temperature of the atmosphere, the emissivity of carbon dioxide decreases logarithmically. This phenomenon is easily observable in the tables on the Total Emissivity of carbon dioxide obtained by means of experimentation by Hottel (1) and Leckner (2) which have been published in many modern texts on Heat Transfer. The following graph illustrates the mentioned negative feedback:



This graph shows the total emissivity of carbon dioxide at increasing partial pressures as well as a comparison between the total emissivity of carbon dioxide against the effective pressure of carbon dioxide. The red bar indicates the total emissivity of carbon dioxide after doubling its partial pressure in the atmosphere (0.0006 atm m).

Doubling the density of carbon dioxide in the atmosphere causes the total emissivity of carbon dioxide decreases, as long as the radiant energy emitted by the surface does not increase causing an increase of

the air temperature; therefore, the total emissivity of carbon dioxide is inversely proportional to its effective pressure and, consequently, to its density in the atmosphere. The same effect has been verified on the tables of total emissivity of carbon dioxide obtained by Hottel, Leckner and other contemporary scientists (1)(2)(3)(4). This fact confirms that carbon dioxide operates as a coolant of the atmosphere and the surface, not as a warmer of the mentioned systems. (6)

Note: For obtaining the pressure of the atmosphere at any altitude, the following formula is applied:

$$p = p_0 * e^{-h/h_0}. \quad \text{(Formula 4)}$$

In Earth, $p_0 = 1 \text{ atm}$, and $h_0 = 7 \text{ Km}$.

For example, if the concentration of carbon dioxide were homogeneous in the whole volume and the pulling of the Earth's gravity on the mass of air didn't change with the amplification of the radius of the mass of air, at 10 meters of altitude, the partial pressure of carbon dioxide would be 0.0003395 atm; at 100 m of altitude, the p_{CO_2} would be 0.000335 atm; at 1000 m of altitude, the p_{CO_2} would be 0.00029 atm, and at 7000 km of altitude, the p_{CO_2} would be 0.00014 atm.

However, the concentration of carbon dioxide decreases with altitude and its density is not the same from one mass of air to another; therefore, the partial pressure of carbon dioxide above 10 meters of altitude is by far lower than the calculated partial pressure by the explained formula in this note. From this observation, it is evident that the practice of multiplying the partial pressure of carbon dioxide measured at an altitude of one meter by the total length of the column of air is a flawed incoherent technique.

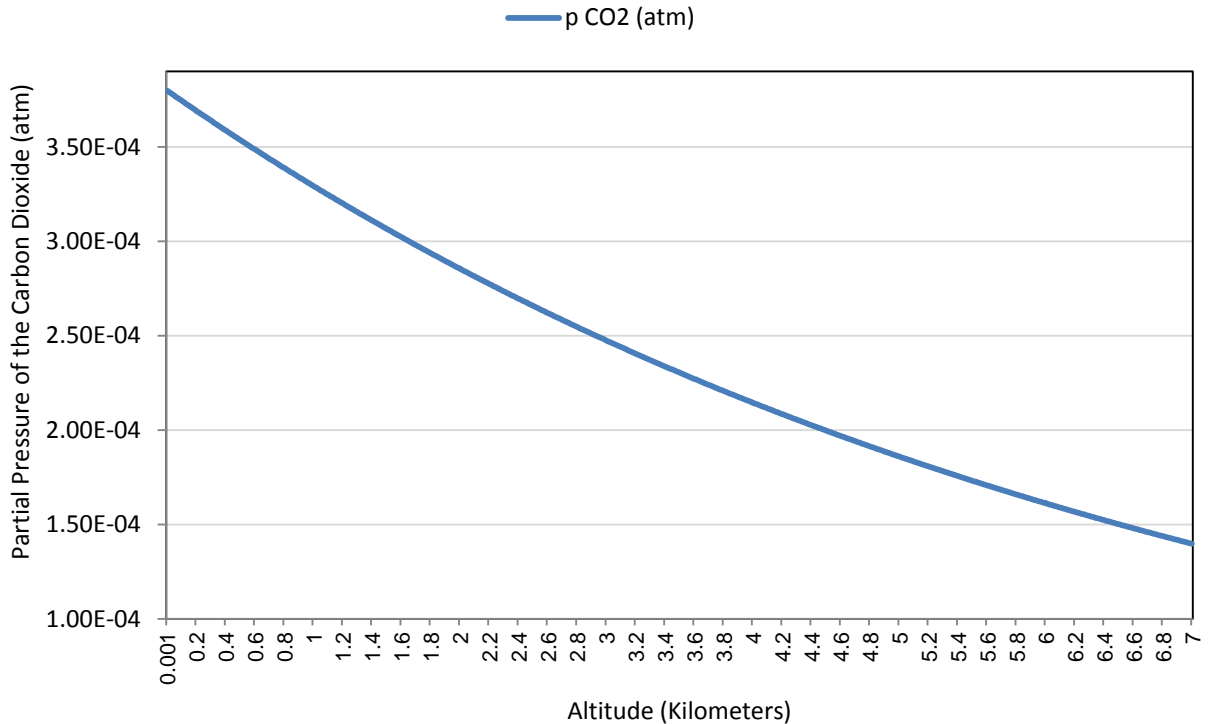
For obtaining the partial pressure of carbon dioxide at its maximum pressure, i.e. at 0 m above sea level, we use the following formula:

$$p_{CO_2} = p_{abs} * \% CO_2 / 100 \quad (1 \text{ atm} = 1.03 \text{ Kg/cm}^2). \quad \text{(Formula 5)}$$

$$p_{CO_2} = 1.03 \text{ Kg cm}^2 * 0.038\% / 100\% = 0.00039 \text{ Kg/cm}^2 = \mathbf{0.00038 \text{ atm}}.$$

The following graph illustrates the change of partial pressure of carbon dioxide with altitude:

Partial Pressure of the CO₂ at $\Delta L = 10 \text{ m} - 7000 \text{ m}$
 (Important Note: The decrease of the density of the CO₂ with the altitude was not considered here; therefore, the partial pressures of the CO₂ at altitudes above 10 m appear higher than its actual partial p



Now I will proceed to obtain the change of temperature caused by carbon dioxide in the atmosphere at current conditions.

To obtain the change of temperature caused by carbon dioxide in the atmosphere, we need to know the energy flux in the whole system as energy is radiated from the surface to the atmosphere:

Energy Flux (Power)

To obtain the energy flux between the surface and carbon dioxide in the air, we apply the following formula:

$$Q = E_{CO_2} A (\sigma) (T_s^4 - T_a^4) \tag{Formula 6}$$

Q = energy flux (power)

E_{CO_2} = Total emissivity of carbon dioxide (from previous algorithm: $E_{CO_2} = 0.0017$)

A = Area = 1 m^2

σ = Stefan-Boltzmann constant = $5.6697 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$

T_s^4 = Temperature in Kelvin of the surface to the fourth power

T_a^4 = Temperature in Kelvin of the air to the fourth power

Known magnitudes:

$$E_{CO_2} = 0.0015$$

$$\sigma = 5.6697 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$$

$$T_s^4 = (330 \text{ K})^4 = 11859210000 \text{ K}^4$$

$$T_a^4 = (308 \text{ K})^4 = 8999178496 \text{ K}^4$$

Introducing magnitudes:

$$Q = E_{CO_2} A (\sigma) (T_s^4 - T_a^4) = 0.0017 * 1 \text{ m}^2 (5.6697 \times 10^{-8} \text{ W/m}^2 \text{ K}^4) (11859210000 \text{ K}^4 - 8999178496 \text{ K}^4) = \mathbf{0.2756 \text{ W}; \text{ rounding up the number, } Q = 0.28 \text{ W}}$$

The energy flux occurs every second; therefore, the energy flux is **0.28 J/s**

The load of energy would be $0.24 \text{ J/s} * 1 \text{ s} = \mathbf{0.28 \text{ J}}$.

In the next step, we need to obtain the normal intensity of radiative heat transfer from carbon dioxide to the surroundings (energy emitted by carbon dioxide by radiation) at its current concentration in the atmosphere at an air temperature of 308 K.

To obtain the normal intensity (I) of radiative heat transfer, we apply the following formula:

$$I = E_{CO_2} (\sigma) (T)^4 / \pi \tag{Formula 7}$$

E_{CO_2} = Total emissivity of carbon dioxide

σ = Stefan-Boltzmann constant = $5.6697 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$

T^4 = Temperature in Kelvin to the fourth power

π = pi, or 3.1415...

Known magnitudes:

$$E_{CO_2} = 0.0017$$

$$\sigma = 5.6697 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$$

$$T^4 = (308 \text{ K})^4 = 8999178496 \text{ K}^4.$$

$$\pi = \text{pi, or } 3.1415\dots$$

Introducing magnitudes:

$$I = E_{CO_2} (\sigma) (T)^4 / \pi$$

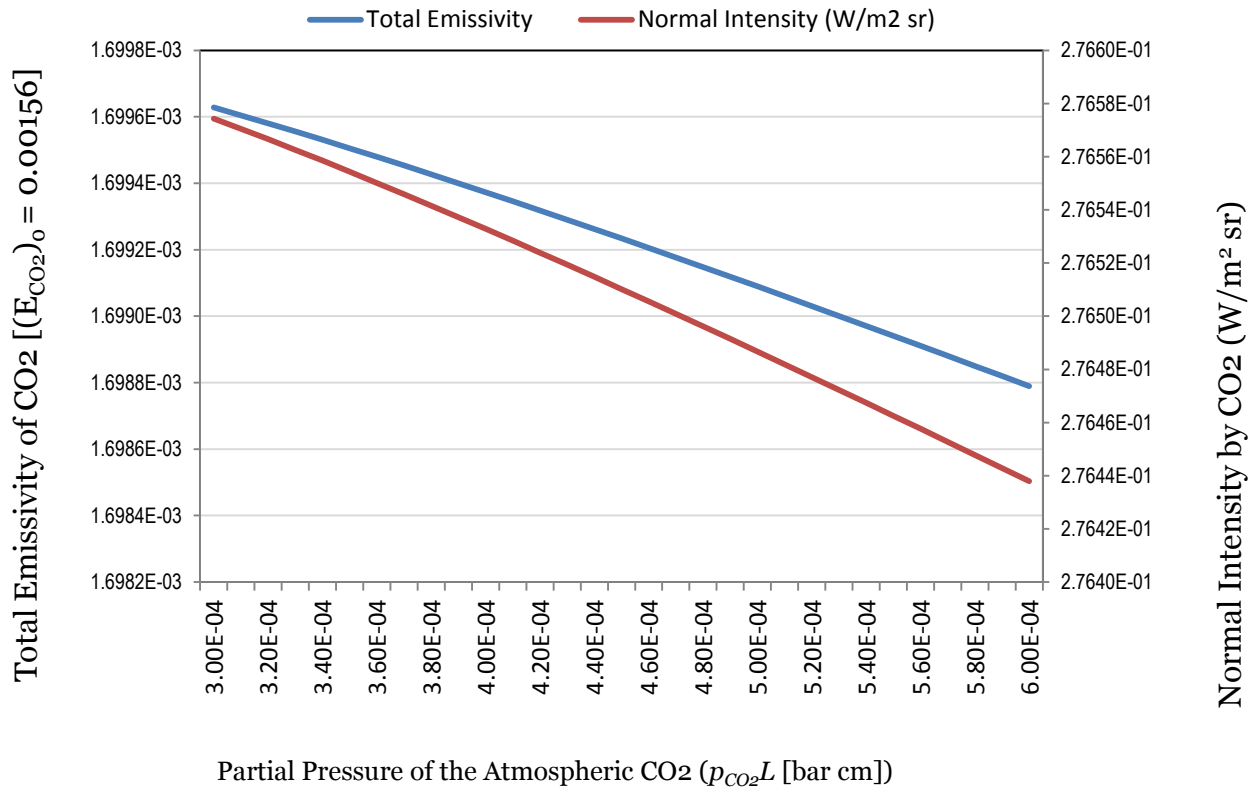
$$I = [0.0015 (5.6697 \times 10^{-8} \text{ W/m}^2 \text{ K}^4) (8999178496 \text{ K}^4)] / 3.141592\dots = 0.765 \text{ W/m}^2 / 3.1416 = \mathbf{0.2761 \text{ W/m}^2 \text{ sr}}$$

sr; rounding up the number, **I = 0.28 W/m² sr**.

Consequently, the total normal intensity (I) of the flux of energy emitted by carbon dioxide in the atmosphere is only **0.28 W/m² sr**. It is not possible for this energy to increase as if by magic. Remember that energy cannot be created or destroyed, but only transformed from one form to another.

The following graph shows the total normal intensity of the radiative heat transfer of carbon dioxide at different partial pressures and their corresponding total emissivities. Notice that the total normal intensity of the radiated energy by carbon dioxide decreases as its mass fraction in the atmosphere increases.

Normal Intensity of Radiative Heat Transfer by Carbon Dioxide



Let us now proceed to obtain the change of temperature caused by carbon dioxide at its current concentration in the terrestrial atmosphere.

The change of temperature caused by the CO₂ at its current concentration in the atmosphere, with the temperature of the air at 308 K:

To obtain the change of temperature caused by the CO₂ in the atmosphere at its current concentration we must apply the following formula:

$$\Delta T = Q / m (Cp) \tag{Formula 8}$$

ΔT = Change of temperature

Q = Amount of thermal energy in transit from the surface to the air

m = mass of carbon dioxide per cubic meter (taken from its current density which is 0.00069 Kg/m³)

Cp = Heat Capacity of carbon dioxide

Known values:

$Q = 0.28$ J (from the formula to obtain the energy flux)

$$m = 0.00069 \text{ Kg}$$

$$C_p = 871 \text{ J/Kg K at 1 atm and 308 K.}$$

Introducing magnitudes:

$$\Delta T = Q / m (C_p)$$

$$\Delta T = 0.28 \text{ J} / 0.00069 \text{ Kg} (871 \text{ J/Kg K}) = \mathbf{0.46 \text{ K.}}$$

1 unit in the Kelvin scale is equivalent to 1 unit in the Celsius scale, so we can express the result as $\Delta T = \mathbf{0.46 \text{ }^\circ\text{C}}$.

By way of contrast, the absolute temperature considered here was 308 K, so the absolute temperature minus the change caused by carbon dioxide gives a difference of 307.5 K.

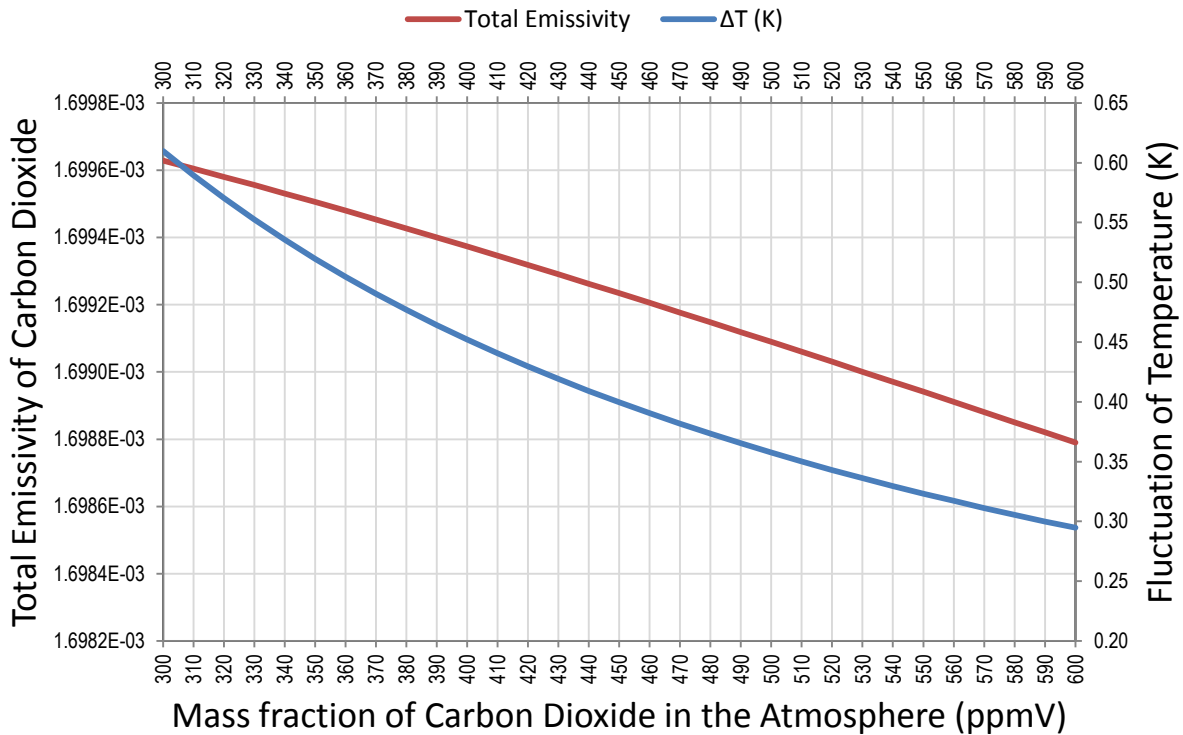
So what the cause of the absolute temperature investigated was? Surely, an increase of the concentration of carbon dioxide in the atmosphere had nothing whatsoever to do with it.

From Leckner ⁽²⁾ tables, we infer the E_{CO_2} at $T = 374 \text{ K}$ and $p_{CO_2} = 0.03 \text{ bar cm}$ is 0.0016. Applying the formula 3, we obtain a total emissivity of the atmospheric carbon dioxide of 0.0017.

The value of Total Emissivity of carbon dioxide, derived from the experimentation by Leckner, in close proximity to the real physical conditions of the atmosphere, is 0.0016 at 374 K and 1 atm of Pabs, so I used this value as a reference for calculating other parameters and proportionality constants.

The following graph shows the fluctuation of temperature of the air caused by carbon dioxide at different mass fractions. Notice that by doubling the mass fraction of carbon dioxide the fluctuation of the temperature would decrease by ca. 4 K, e.g. if the temperature of the air is 35 °C, by doubling the mass fraction of carbon dioxide the temperature of the air decreases down to 31 °C. Therefore, the conclusion is that carbon dioxide operates like a coolant in the atmosphere.

Total Emissivity of Carbon Dioxide at Different Mass Fractions and Corresponding ΔT by CO_2



MODELS FROM ABSTRACT DATA

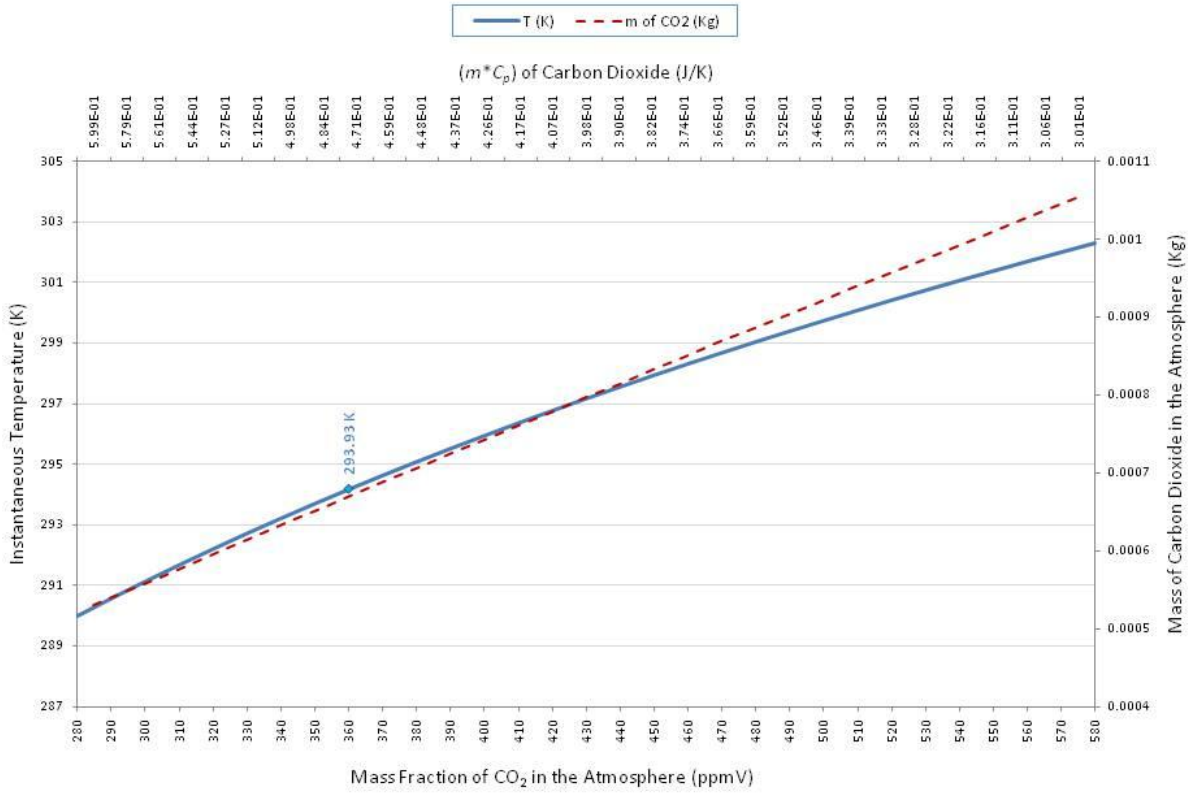
When we introduce abstract data in our databases and somehow we force the magnitudes for obtaining preconceived results, the physics becomes absolutely speculative.

In the following graph, I have modeled the absolute temperature that carbon dioxide would hypothetically produce by theoretical increases of its density and temperature.

The blue solid line describes the hypothetical absolute temperature caused by the fluctuations of carbon dioxide density and temperature. The indigo dotted line denotes the mass of carbon dioxide in the atmosphere. The marker indicates the temperature that speculatively the increase of atmospheric carbon dioxide would cause at the current conditions.

Notice that the variation of temperature caused by 280 ppmV of carbon dioxide would be 0 K, i.e. the temperature of the atmosphere would increase up to 290 K (17 °C, or 62.6 F). Observe also that the differential temperature at the real present conditions, with respect to the standard instantaneous temperature, would be 5.1 K, supposedly caused solely by carbon dioxide, which is wholly missing in the natural world.

Modeled T_{abs} Caused by Increasing Density and Temperature of the CO_2



Graph on Projected Instantaneous Temperature by Increasing Mass and Temperature of the CO_2 in the atmosphere. ©2010 by Nasif Nahle

According to this model, if we doubled the density of carbon dioxide in the atmosphere, the temperature would be increased by 12.3 K. The latter would mean that if the temperature of the atmosphere is 310 K (37 °C or 98.6 F), carbon dioxide alone would make the temperature of the air increased up to 322.3 K (49.3 °C or 120.74 F), which evidently is not possible in real nature.

Perhaps you have noticed that the partial pressure of carbon dioxide and the energy absorbed by the surface were not taken into account to elaborate this model, and this practice is flawed.

CONCLUSION

Carbon dioxide does not contribute appreciably with the greenhouse effect. The contribution of carbon dioxide to the current anomaly of temperature at its current concentration in the atmosphere is really insignificant.

The failure of carbon dioxide for causing a large change of temperature in the atmosphere obeys to its intrinsic physical properties, not to negative feedbacks triggered by other more efficient greenhouse gases in the air than carbon dioxide, i.e. water vapor, methane, nitrogen dioxide, and carbon monoxide.

The calculations on this investigation show that the total emissivity of carbon dioxide decreases as the density of the gas in the atmosphere increases, so we should expect that at higher concentrations in the atmosphere, carbon dioxide could act as a coolant of the atmosphere and the surface of the Earth as long

as the energy emitted by the surface does not increase. If the latter happens, the total emissivity of carbon dioxide would increase, so its contribution to the greenhouse effect would increase.

The inversely proportional correlation between the total emissivity of carbon dioxide and its density in the atmosphere may obey to an increase of more available microstates toward which the energy emitted by the surface and other internal systems in the atmosphere is transferred by radiation.

Carbon dioxide emitted by human activity cannot be the cause of climate change as it is incapable physically of causing a significant anomaly of the atmospheric temperature.

Any assertion—involving the physics of radiative heat transfer—that carbon dioxide is a causative agent of climate change, is a deliberate pseudoscientific misrepresentation.

FURTHER READING

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